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Dynamical 'breaking' of time reversal symmetry

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Abstract
It is a common assumption that quantum systems with time reversal invariance and classically chaotic dynamics have energy spectra distributed according to GOE type of statistics. Here we present a class of systems which fail to follow this rule. We show that for convex billiards of constant width with time reversal symmetry and 'almost' chaotic dynamics the energy-level distribution is of GUE type. The effect is due to the lack of ergodicity in the 'momentum' part of the phase space and, as we argue, is generic in two dimensions.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The famous conjecture of Bohigas, Giannoni and Schmit (BGS) [1] asserts that the energy levels of classically chaotic systems are distributed as eigenvalues of random matrix ensembles. Accordingly, the statistics of the energy levels is universal and depends only on symmetries of the system. This means the energy levels distribution for (spinless) chaotic systems with time reversal invariance should be close to that of Gaussian orthogonal ensemble (GOE). If the time reversal invariance is broken the distribution of the energy levels follows statistics of Gaussian unitary ensemble (GUE). The BGS conjecture has been supported by broad numerical and experimental evidence. Indeed, for a large number of systems without additional symmetries the spectral statistics are in agreement with the above predictions. This might not be true, however, if additional symmetries are present. Examples of symmetric billiards with anomalous spectral statistics were given in [2, 3]. These billiards are time reversal invariant, but in addition have some rotational symmetry. As a result, the statistics for a part of their spectra turns out to be of GUE type (rather than of GOE type). Furthermore, anomalous spectral statistics also appear in systems with broken time reversal invariance. For example, the energy levels of magnetic billiards with reflection symmetry are known to be distributed as in GOE [4]. Also, Poisson-like spectral statistics, characteristic of integrable systems, are

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known to exist in completely chaotic arithmetic billiards on surfaces of the constant negative curvature [5]. Here, the anomaly can be traced to the existence of very large multiplicities in the length spectrum of the classical periodic orbits. Note that, on the other hand, in a special regime GOE and GUE-type statistics might appear in some integrable models [6].

In this paper, we introduce a class of convex billiards of constant width with smooth boundaries whose dynamics are time reversal invariant and 'almost' chaotic. These billiards have no additional symmetries (neither explicit nor hidden) and, in particular, have no anomalous degeneracies in the length spectrum of the periodic orbits. Nevertheless, the corresponding quantum billiards exhibit spectral statistics of GUE type. Furthermore, if an additional reflection symmetry is present in the systems, the spectral statistics turns out to be of GOE type. In the body of the paper, we give an elementary semiclassical explanation for these results and discuss the implications for spectra of generic convex billiards with smooth boundaries.

2. Billiards of constant width

We deal with a class of convex billiard tables Ω of constant width. This means, for any point x at the boundary $\partial\Omega$ the maximal distance between x and other points of $\partial\Omega$ is a constant independent of x. There is a simple way to construct such domains by means of the following parameterization of the domain's boundary in the complex plane [8]. The curve,

$$z(\alpha) = z(0) - \mathrm{i}\sum_{n\in\mathbb{Z}} \frac{a_n}{n+1}(\mathrm{e}^{\mathrm{i}\alpha(n+1)} - 1) \in \mathbb{C}, \qquad \alpha \in [0, 2\pi), \tag{1}$$

defines the boundary of the domain of the constant width $2r$ and perimeter $2\pi r$, whenever: (1) $z(\alpha)$ has non-negative curvature; (2) the parameters a_n's satisfy conditions, $a_0 = r$, $a_{-n} = a_n^*$, $a_1 = 0$, $a_{2n} = 0$ for $n > 0$. Since the first condition is automatically satisfied if, for instance, a_0 is sufficiently large, the parameterization (1) provides, in fact, a large family of billiards of constant width with smooth boundaries. Note also that using (1) one can obtain shapes with additional space symmetries. In particular, Ω has a reflection symmetry if all a_{2n+1}, $n \in \mathbb{Z}$ are either purely imaginary or real numbers.

Previously, billiards of constant width have attracted attention due to their unusual geometric properties (see, e.g., [8] for interesting properties of their caustics). Our interest here stems from their peculiar dynamical behaviour: the billiard ball hitting the boundary of the billiard at an angle in the interval $[0, \pi/2)$ (resp. $[\pi/2, \pi)$) must hit the boundary next time at an angle in the same interval. In other words, the billiard ball once launched clockwise (resp. anti-clockwise) will move in that way forever.

The billiard dynamics can be described in a standard way with the help of the associated Poincare map. The map acts on unit vectors attached to the boundary by translating them according to the rules of billiard dynamics. The corresponding two-dimensional phase space can be parameterized by a couple of coordinates prescribing position and direction of the unit vectors. The canonical choice is $(s, \cos\theta)$, where $s \in [0, 2\pi a_0)$ is the arclength parameter along the boundary and $\theta \in [0, \pi]$ is the angle between the unit vector and the tangent line to the boundary. The phase space of any system with time reversal invariant dynamics has the reflection symmetry along the line $\theta = \pi/2$. For billiards of constant width this symmetry line is, furthermore, invariant under the billiard map and separates the motions in the clockwise and anti-clockwise directions, see figure 1.

In general, a billiard of constant width defined by equation (1) has a mixed phase space where regions of regular motion, i.e. Kolmogorov–Arnold–Moser (KAM) tori and elliptic islands coexist with regions of chaotic motion. In particular, in the vicinity of the line

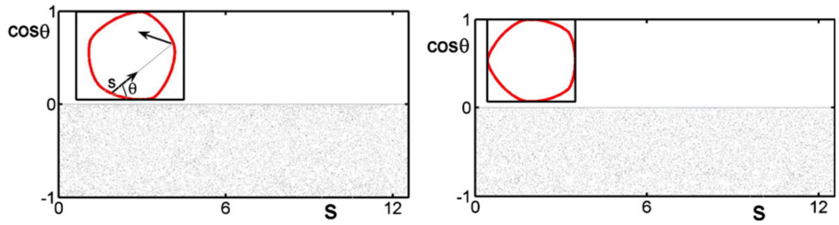

Figure 1. The insets show non-symmetric (left) and symmetric (right) billiards of constant width with parameters ($a_0 = 2, a_3 = \mathrm{i}/4, a_5 = 1/2 + \mathrm{i}/2, a_{2k+1} = 0, k > 2$) and ($a_0 = 2, a_3 = \mathrm{i}/4, a_5 = -3\mathrm{i}/4, a_{2k+1} = 0, k > 2$) respectively. The corresponding phase-space pictures are obtained after hundreds of iterations of the billiard map applied to a number of initial points located in the lower ($\theta > \pi/2$) half of the phase space. Note that the iterated points do not penetrate into the upper half. This illustrates the separation of the clockwise and anti-clockwise types of motion. For a comparison with a generic convex billiards, see figure 5.

$\theta = \pi/2$ there always exists a region filled by KAM tori. For our purposes, it will make sense to consider billiards with 'maximally' chaotic phase space. Two such billiards are shown in figure 1. Here the parameters a_n are adjusted in a way to minimize the sizes of elliptic islands as well as regions of KAM tori along the lines $\theta = \frac{\pi}{2}$ and $\theta = 0, \pi$ (whispering gallery region). In particular, whispering gallery KAM tori can be completely destroyed by forcing the curvature of $\partial\Omega$ to vanish at some point.

3. Spectral statistics

We shall consider the energy spectrum of the quantum billiards in figure 1, subjected to the Dirichlet boundary conditions at $\partial\Omega$. Denote by $\mathcal{H} = -\frac{1}{2}\hbar^2\Delta$ the corresponding Hamiltonian. As already noted, the change from clockwise to anti-clockwise types of motion is a classically forbidden process. On the other hand, in the quantum billiards such a switch is possible due to the tunnelling effect. This leads to the quasidegenerate structure $\{E_n^s, E_n^a\}$ for the most part of the spectrum of $\mathcal{H}$. Here, the pairs of energy levels E_n^s, E_n^a correspond to the symmetric $\varphi_n^s \approx (\varphi_n^+ + \varphi_n^-)$ and the antisymmetric $\varphi_n^a \approx (\varphi_n^+ - \varphi_n^-)$ combinations of the clockwise φ_n^+ and the anti-clockwise φ_n^- quasimodes whose Wigner transforms are entirely concentrated in the lower ($\theta > \pi/2$) V^- and the upper ($\theta < \pi/2$) V^+ halves of the phase space V of the billiard flow. The splittings $\delta E_n = |E_n^a - E_n^s|$ are determined by the tunnelling time $\tau \sim \hbar/\langle\delta E_n\rangle$ needed to pass the dynamical 'barrier' $\theta = \frac{\pi}{2}$. Since τ is exponentially large in $\hbar$, this results in exponentially small splittings δE_n between the energy levels. The rest of the spectrum contains unpaired zero angular momentum bouncing modes localized in the phase space exactly on the separation line $\theta = \frac{\pi}{2}$.

The above crude argument can be turned into a rigorous one along the following lines. Take $\Gamma_+ \subset V_+, \Gamma_- \subset V_-$ as a pair of symmetric KAM tori near the separation line $\theta = \pi/2$ and consider the domains $D_+ \subset V_+, D_- \subset V_-$ which are the parts of V_+ (resp. V_-) bounded by Γ_+ (resp. Γ_-). For such domains one can construct approximate quantum projection operators π_+, π_-, whose classical symbols are just characteristic functions on D_+ and D_-, see, e.g., [9]. Since D_+, D_- are invariant domains under the classical flow, $\pi_\pm$ commute with the Hamiltonian up to some order N in $\hbar$

$$[\pi_\pm, \mathcal{H}] = O(\hbar^N). \tag{2}$$

Moreover, for invariant domains whose boundaries are composed of KAM tori, there exists construction of $\pi_\pm$ satisfying (2) for an arbitrarily large N [9, 10]. The projection operators $\pi_\pm$

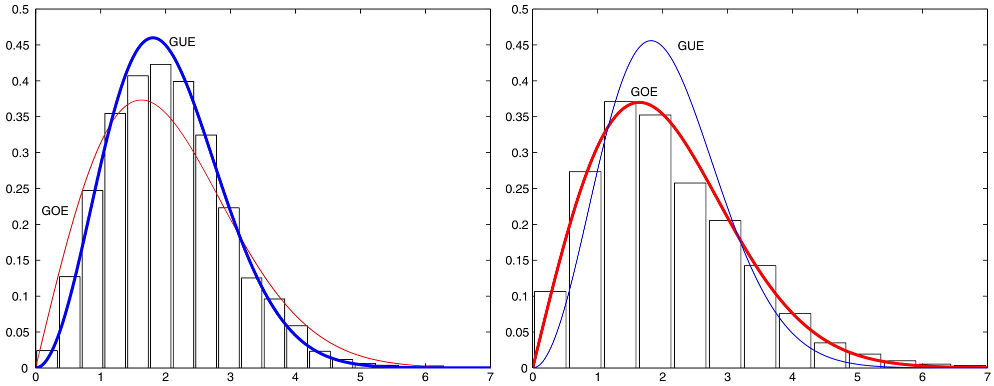

Figure 2. Nearest neighbour distribution of energy levels for the non-symmetric (left) and symmetric (right) billiards in figure 1. For comparison, exact GOE and GUE distributions are shown as well.

can then be utilized to decompose an eigenfunction φ_n of $\mathcal{H}$ into clockwise and anti-clockwise moving quasimodes

$$\varphi_n^+ = \pi_+\varphi_n, \qquad \varphi_n^- = \pi_-\varphi_n. \tag{3}$$

As follows immediately from (2) both φ^+, φ^- provide approximate solutions of the Schrödinger equation

$$\mathcal{H}\varphi^\pm = E_n\varphi^\pm + O(\hbar^N). \tag{4}$$

Since φ^+, φ^- are quasiorthogonal to each other their linear combination approximates (under some assumptions on spectral degeneracies) two real quasidegenerate eigenfunctions φ_n^s, φ_n^a of $\mathcal{H}$, see, e.g., [12]

$$\varphi_n^s = \varphi^- + \varphi^+ + O(\hbar^\infty), \qquad \varphi_n^a = \varphi^- - \varphi^+ + O(\hbar^\infty), \qquad \delta E_n = \left|E_n^a - E_n^s\right| = O(\hbar^\infty), \tag{5}$$

where we have assumed $N = \infty$. The first of these eigenfunctions is just $\varphi_n \equiv \varphi_n^s$ itself while the second one is its antisymmetric counterpart. Note that up to now, the choice of Γ_+, Γ_- has been somewhat arbitrary. Now, let us choose $\Gamma_\pm$ to be the outermost KAM tori which separate chaotic parts D_+, D_- of the phase space from the regular part $D_0 = V\backslash D_+ \cup D_-$. Let π_0 be the projection operator on D_0. Then by using all three projection operators one can separate the eigenstates of $\mathcal{H}$ into quasidegenerate pairs of 'chaotic' eigenstates localized in $D_+ \cup D_-$ and 'regular' eigenstates $\left\{\varphi_n^0\right\}$ localized in D_0 [9]. The proportion of each type of the states is determined by the Liuville measure of the corresponding invariant domain. Hence, for the billiards in figure 1, the regular energy levels $\left\{E_n^0\right\}$ constitute a tiny fraction of the whole spectrum. (Actually, for the considered range of energies only unpaired bouncing modes appear.)

Some of the previous studies on the so-called Shnirelman peak (although for different, quasi-integrable systems) [11] have been concentrated on the behaviour of quasidegeneracies δE_n. Here we are rather interested in the proper statistics of the levels E_n^s, E_n^a. To this end, we have numerically calculated by the scaling method of Vergini and Saraceno [13] a number ($\sim$15 000) of energy levels for each of the billiards in figure 1. Since practically almost all levels are paired it makes sense to consider half of the spectra, e.g. E_n^s. The results for the nearest-neighbour distribution $P(s)$ of E_n^s are presented in figure 2. As one can clearly see

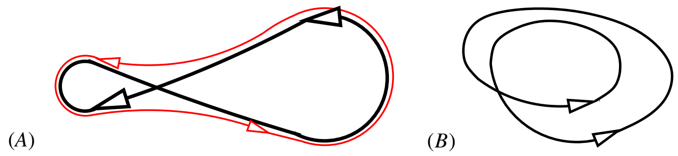

Figure 3. (*A*) Sketch of a pair of self-encountered periodic orbits. (*B*) A 'typical' periodic orbit in a billiard of constant width. Note that pairs of self-encountered periodic orbits do not appear in billiards of constant width.

the distribution for the billiard without additional symmetries (left in figure 1) clearly follows the pattern of GUE. This contradicts a common belief that chaotic systems with time reversal invariance have spectra of GOE type when additional symmetries are absent. In contrast, the distribution $P(E)$ for the billiard with a reflectional symmetry (right in figure 1) exhibits GOE type of statistics. Below we provide an elementary explanation for these results based on the semiclassical link between the spectral statistics and the periodic orbits of the system. Specifically, let us focus on the spectral form factor $K(T)$. It is defined as the Fourier transform of the autocorrelation function

$$R(s) = \bar{d}^{-2}\langle d(E+s)d(s)\rangle - 1, \tag{6}$$

where $d(E) = \sum \delta(E - E_n), \bar{d} = \langle d(E)\rangle$ denote the density of states and its mean value respectively. By means of the semiclassical trace formula the density of states can be written as a sum $d(E) = \bar{d} + \sum A_n \exp(\mathrm{i}S_n(E)/\hbar)$ over periodic orbits, where phases $S_n(E)$ include both actions and Maslov indices of the periodic orbits. After the substitution of $d(E)$ into (6) and taking the Fourier transform one gets the semiclassical representation of $K(T)$ as double sum over pairs of periodic orbits. The spectral form factor can be naturally separated $K(T) = K_{\mathrm{diag}}(T) + K_{\mathrm{off}}(T)$ into two terms provided by diagonal ($S_i = S_j$) and off-diagonal ($S_i \neq S_j$) correlations of periodic orbits.

The leading diagonal term was derived by Berry [14] and in the Heisenberg time $T_H = 2\pi\hbar\bar{d}$ units $t = T/T_H$ found to be $K_{\mathrm{diag}}(t) = \beta t$, with $\beta = 2$ for time reversal invariant systems, and $\beta = 1$ otherwise. This should be compared with the spectral form factors

$$K_{\mathrm{GUE}}(t) = t, \qquad K_{\mathrm{GOE}}(t) = 2t + t\ln(2t+1), \quad (t < 1)$$

for GUE and GOE respectively. In the absence of time invariance K_{off} vanishes and the diagonal term alone reproduces K_{GUE} correctly. In contrast, for time reversal invariant systems K_{diag} gives only leading term and the off-diagonal correlations between periodic orbits must provide the rest. It was, indeed, shown by Sieber and Richter [15] that the GOE result can be reproduced correctly if one takes into consideration the correlations between pairs of self-encountered periodic trajectories which approach themselves from the opposite directions under small angles. More specifically, the nth-order term in the Taylor expansion of $K_{\mathrm{GOE}}(t)$ comes from the correlations of pairs of periodic orbits with $n-1$ self-encounters. It is a straightforward observation that pairs of self-encountered periodic orbits just do not exist in billiards of constant width, since trajectories cannot reverse their directions of motion, see figure 3. This implies that $K_{\mathrm{off}}(t)$ must be zero and $K(t) = K_{\mathrm{GUE}}(t) \equiv t$. Hence, the spectral form factor of the non-symmetric billiard in figure 1 (left) should be of GUE and not of GOE type. On the other hand, for the billiard in figure 1 (right) the reflection symmetry substitutes the role of time reversal invariance and restores correlations between periodic orbits. The simplest way to see this is to consider a half of the billiard. The dynamics there are not

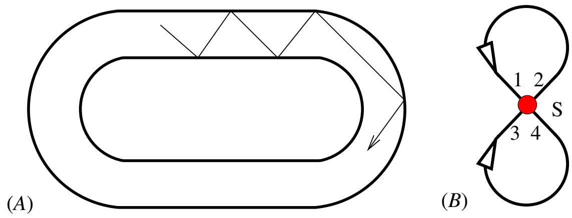

Figure 4. On the left (A): fully chaotic 'hippodrome' billiards. Both the external and internal billiard walls are composed of the boundaries of stadia such that the corridor in-between has a constant width. On the right (B): 'unidirectional' quantum graph. The scattering matrix at the vertex satisfies conditions: $S_{1,3} = S_{3,1} = S_{2,4} = S_{4,2} = 0$, $S_{i,i} = 0$, $i = 1, \dots, 4$.

unidirectional and self-encountered trajectories do exist. This leads back to GOE type of spectral statistics. This is in complete analogy with the case of reflection symmetric billiards in the presence of magnetic field, where one observes GOE statistics instead of GUE [4].

The billiards considered so far are only 'approximately' chaotic. But there do also exist fully chaotic billiards of constant width in multiply connected domains. An example of such like billiards is shown in figure 4. The quantum billiards of this type (Monza billiards) have been recently investigated by Veble, Prosen and Robnik [7]. As in the case of convex billiards of constant width, the clockwise and anti-clockwise types of motion are completely separated. Here, however, the separation is sharp: there are exactly two ergodic components, and as can be rigorously proven for the billiard in figure 4, each of them is fully hyperbolic with a positive Lyapunov exponent almost everywhere. This gives rise to significant differences in the spectral properties of the corresponding quantum systems. For fully chaotic billiards the width of the 'barrier' between two types of motion shrinks to zero and the tunnelling time is determined by diffraction effects at the points of billiard boundary with curvature jumps. This results in much shorter (algebraic rather than exponential in $\hbar$) tunnelling times. Thus, instead of exponentially small quasidegeneracies, one gets splittings between the energy levels comparable with the mean-level spacing. (Alternatively, one can note that N in (2) must be finite.) Unlike the case of billiards with smooth boundaries, here it is not even clear, whether the eigenfunctions can be actually separated into quasidegenerate pairs. Also, note that diffractional periodic trajectories hitting the boundaries at the points with curvature jumps can switch their directions and correlations among them are possible. Hence, in this case it seems plausible that K_{off} does not vanish entirely and the overall spectral statistics are not purely of GUE type. Nevertheless, it has been numerically shown in [7] that the long-range correlations among levels tend to exhibit GUE-type behaviour. This is in agreement with our results for convex billiards of constant width.

4. Generic convex billiards

Let us consider now the implications of the above results for spectra of generic billiards whose dynamics is neither fully integrable nor chaotic. In that case, by the Berry–Robnik theory [16] the energy spectrum is composed of the independent spectra corresponding to the invariant parts of the phase space. Thus for systems with time reversal symmetry, one might expect the energy-level distribution to be a mixture of Poissonian statistics with GOE-type statistics related to the regular and chaotic dynamics, respectively. However, as we argue below, a general picture should be somewhat different. Call an invariant ergodic component D of the

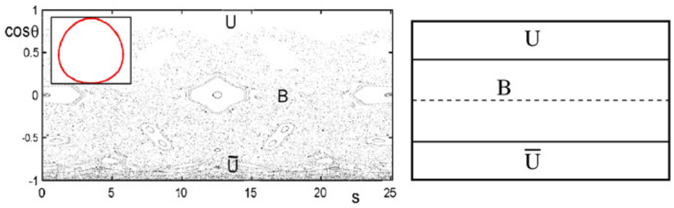

Figure 5. On the left is shown a 'generic' ($a_2 \neq 0$) billiard defined by (1) with parameters $\{a_0 = 4, a_2 = 0.1, a_3 = 0.5, a_5 = 0.1, a_k = 0, k > 5\}$. The billiard map is applied to the initial points located in the lower half of the phase space. Note that the iterated points do not penetrate into a certain domain U. Hence, U is dynamically separated from the symmetric domain $\bar{U}$ and the corresponding dynamics are unidirectional. In contrast, the dynamics in the central part B of the phase space are bidirectional. On the right is depicted the caricature of the phase-space structure of generic convex billiards in two dimensions.

phase space *time reversal connected* if for every point $(q, p) \in D$ with the coordinate q and momentum p, the 'reverse' point $(q, -p)$ belongs to D, too. Now consider an invariant region D_i with chaotic dynamics. If D_i is time reversal connected, the dynamics inside the domain are bidirectional. This means the billiard ball launched from such a region might reverse the direction of the flight in the course of the motion and return to the vicinity of the starting point with the opposite momentum. As a result, the periodic trajectories admit self-encountering and the corresponding spectral statistics are of GOE type. In contrast, if D_i is not time reversal connected, the motion inside of it is always unidirectional. Here the full switch of flight direction combined with the simultaneous return in the space is a dynamically forbidden process (e.g. due to the existence of separating KAM tori) and the resulting statistics should be of GUE type with quasidegeneracies. For a typical convex billiard with smooth boundaries in two dimensions the chaotic part of the phase space contains both types of regions, as shown in figure 5. Thus, in the absence of spatial symmetries the overall spectral statistics must be a mixture of independent GUE, GOE and Poissonian statistics corresponding to the invariant sets with unidirectional, bidirectional chaotic dynamics and regular dynamics. If an additional reflection symmetry exists in the billiard, then only GOE and Poissonian parts are present. In more than two dimensions, however, KAM tori in general do not separate regions of phase space. So it might be expected that typically only bidirectional type of motion exists and only GOE type of subspectra appears.

It is worth noting that a mixture of a large number of independent subspectra would result in the Poissonian statistics, irrespectively of the statistics of individual components. Thus for generic systems, it would be hard, in practice, to observe the appearance of GUE-type subspectra. This probably explains why the effect has been, by and large, overlooked so far. The billiards of constant width represent a very special class of dynamical systems where bidirectional type of dynamics is completely absent and the effect can be clearly observed.

5. Conclusions

To summarize, we have presented a wide class of time reversal quantum billiards with an anomalous spectral behaviour. By virtue of their unidirectional dynamics, one can clearly observe a general pattern of spectral behaviour in classically chaotic systems, which would be hard to see otherwise. Namely, in the absence of additional symmetries, time reversal invariance of chaotic systems does not automatically guarantee GOE-type statistics for the energy spectrum. In addition, a dynamical condition must be satisfied. A chaotic invariant domain exhibits statistics of GOE type only if it is time reversal connected, otherwise the corresponding spectrum is of GUE type.

Note also that billiards of constant width can be actually realized as real experimental devices, e.g. quantum dots, microwave cavities etc. Loosely speaking, time reversal symmetry can be broken here by geometric means (by switching from symmetric to non-symmetric shapes), rather than with external magnetic fields. Besides the spectral statistics, other quantum properties of these systems should be affected too. For instance, the semiclassical treatment of the Landauer conductance in [17] and shot noise in [18] through ballistic devices rely on calculations of the correlations between self-encountered periodic trajectories. So one can use the previous arguments to conclude that conductance, shot noise, etc of quantum dots with non-symmetric (symmetric) shapes of constant width should be as in the systems without (resp. with) real time reversal symmetry.

Finally, it is worth mentioning that besides billiards, there exist other systems with unidirectional type of dynamics. For instance, the present billiard construction using parameterization (1) can be straightforwardly generalized to get a family of smooth 'unidirectional' potentials. Namely, fix the coefficients $a_n, n > 0$, set $z(0) = -\mathrm{i}a_0$ and let a_0 vary over an interval $[\delta_1, \delta_2]$, $\delta_{1,2} > 0$, such that both conditions in (1) are satisfied. This defines a family $\gamma(a_0)$, $\delta_1 \leqslant a_0 \leqslant \delta_2$ of closed convex curves of constant width in the domain of complex plane bounded by $\gamma(\delta_1)$ and $\gamma(\delta_2)$. Now, let v be a smooth potential which is equal ∞ outside $\gamma(\delta_2)$, 0 (or ∞) inside $\gamma(\delta_1)$ and whose equipotential lines coincide with $\gamma(a_0)$, $a_0 \in [\delta_1, \delta_2]$ in between. Any such potential v gives rise to the unidirectional Hamiltonian flow inside the domain bounded by $\gamma(\delta_2)$. Another class of unidirectional systems is provided by quantum graphs of a certain type. A simple example is shown in figure 4. Here the full separation of two types of motion is achieved by putting appropriate scattering matrices at the vertices of the graph.

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